Document ID: 01_19_99_1
Date Received: 1999-01-19 **Date Revised**: 1999-04-07 **Date Accepted**: 1999-04-12
Curriculum Topic Benchmarks: M1.41., M1.4.8, M3.4.1, M8.4.12, M1.3.11, M8.3.11
Grade Level: [9-12] High School
Subject Keywords: social security, algebra, problem-solving process, computation, functions
Rating: moderate

Grandpa's Social Security

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This simple example shows how algebra can be useful in the real world: **Should Grandpa start receiving his Social Security Benefits at age 62 or should he wait until age 65?**

Background: Most retired Americans are eligible to start receiving Social Security benefits at age 62. However, if a retiree waits until age 65 to receive benefits, the monthly check is larger.

Exercise: Determine how long a retiree has to live until the total benefits are the same whether he retires at age 62 or age 65.

Presentation: Set up a situation for the students. Tell each one that his or her grandfather has come to the student and asked if he should take his retirement benefits when he reaches 62 or should he wait until he reaches 65. He explains that he doesn't need the money to live on at present but wants to have the maximum amount of money when he dies to pass on to his grandchildren (meaning you!). He wants to know how long he has to live to make it beneficial to wait until 65 to start taking his Social Security payments.

Explain to each student roughly how Social Security works. The Federal Government keeps track of how much you earn each year of your life. They also collect Social Security payments from you and your employer each year. When you approach retirement, you can have the Social Security Administration tell you how much you will earn each month based on your work history. The base amount is computed assuming you will retire and begin receiving benefits at age 65. If you decide to retire at age 62, this amount is reduced by 20%.

Brute Force Approach: The student might create a table of how much Grandpa will have at the end of each year based on his year of starting the benefits. Let's assume his payments will be $1000 per month at age 65. We assume the students will then know that the payments will be $800 a month if he starts receiving them at age 62. Then the total payments for the year will be 800x12 = $9600 in the one case and 1000x12 = $12,000 in the other case.

End of Year	Start at 62 /year	Total	Start at 65 /year	Total
62	9600	9600	0	0
63	9600	19200	0	0
64	9600	28800	0	0

65	9600	38400	12000	12000
66	9600	48000	12000	24000
67	9600	57600	12000	36000
68	9600	67200	12000	48000
69	9600	76800	12000	60000
70	9600	86400	12000	72000
71	9600	96000	12000	84000
72	9600	105600	12000	96000
73	9600	115200	12000	108000
74	9600	124800	12000	120000
75	9600	134400	12000	132000
76	9600	144000	12000	144000
77	9600	153600	12000	156000
78	9600	163200	12000	168000

So we see that if Grandpa lives to the end of his 76th year, or in other words, to his 77th birthday, then he will have received the same amount of money regardless whether he started receiving at age 62 or age 65. If he lives longer than 77, he does better by delaying the start of his Social Security until age 65.

Have Students Survey Folks on Social Security: Instead of taking the figure of $1000 in the above, have each student talk to someone on Social Security (a grandparent, an aunt, uncle, etc.) and find out what their monthly Social Security payment is and have each student use this figure or, perhaps, take all the amounts and use the mean.

Plot the Two Cases: Have the students plot the above two cases on a plotting calculator or on paper and have them observe where the two lines cross. This is the "break-even" point where Grandpa will do equally well regardless of when he starts taking his benefit.

Use a Symbol for the Yearly Amount: You could now ask the students to redo this table but make the amount Grandpa receives each year equal to the symbol Y. For example:

End of Year	Start at 62 /year	Total	Start at 65 /year	Total
62	0.8Y	0.8Y	0	0
63	0.8Y	1.6Y	0	0
64	0.8Y	2.4Y	0	0
65	0.8Y	3.2Y	Y	Y
66	0.8Y	4.0Y	Y	2Y
….				

Let them determine when the break-even age is reached. Many students will realize right away that it doesn't matter what the amount is, the time to reach the break-even point is the same.

Doing the Problem with Algebra: Make an equation of the situation. Let X be Grandpa's age and let Y be the amount of money received each year. Then, if Grandpa takes his benefit at age 62, the amount he will have received at age X for any age past 62 is: 0.8Y(X - 62)

The 0.8 reflects that he only gets 80% of the benefit he would have received at age 65 if he had waited. If he had waited, the amount received at any age X is: Y(X - 65)

Since we wanted to find at what age these benefits will be the same, set these two expression equal to each other: $0.8Y(X - 62) = Y(X - 65)$

If the students have had algebra, they should recognize that the Y can be divided into both sides of the equation resulting in: $0.8(X - 62) = X - 65$

So, as we said above, the amount received per month does not matter. And solving for X yields:

$$X = 77$$

So, just as above, if Grandpa lives to 77 he gets the same amount.

Wow! This was a lot easier than constructing that table!

What if Grandpa wants to receive his benefits at age 62 and 6 months? The amount that a benefit is reduced is not a flat 20% as stated above. What the Social Security Administration actually does is reduce the amount by 5/9 of one percent for each month before 65 that a person begins receiving a benefit. Ask the class to compute what the break-even rate is for some other starting age such as 62 years and 6 months.

A Complication: All of the above assumes that the payments do not change each year. However, they do change. Each year the government computes the cost of living and adjusts the Social Security payments accordingly. Ask the student how the above would change if there was a three percent cost of living adjustment each year.

A Further Complication: What if Granddad is very clever and figures he can earn 10% per year by taking this payment and investing it. So, with a cost of living adjustment of 3% and an assumed 10% return on investments, now what age does he have to live to for a break-even situation?

What is Grandpa's life expectancy? Another avenue of exploration would be to ask the students what Grandpa's life expectancy is knowing that he has lived to age 62. Help them find tables that give the average life expectancy based on how long a person has lived. They should appreciate the fact that a person's life expectancy at birth is quite different from their life expectancy at other ages.

And What About Grandma? The students will find in looking up actuarial tables that women have a longer life expectancy than men. (They probably know this already; they probably don't know the numbers.) What should Grandma do about her social security? Grandma has the option of taking any social security she has earned or of taking one-half of the amount the Grandpa gets. (If the amount she is entitled to is more than Grandpa's, then their roles are reversed and Grandpa has the option of taking his or one-half of the amount that Grandma gets.) If Grandpa dies first, she continues to receive the same amount.