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Corrupted Politicians

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You can find statistics in every day life. Counting problems arise in situations where selections are made as in the following example of corrupted politicians.

Formulation of the problem

The town council of a certain city consists of members of 3 political parties (A, B and C), But not all of them could resist the trap of corruption: 5 members of the council received a bribe. The chairman of party B is disturbed and wants to estimate the fairness among his party members. Please help him and calculate the chance that there are at least 2 members of party B among the corrupted council members?
You can make the assumption that all members of the council have equal chances to receive a bribe and that once they get offered a bribe, they will accept it. The composition of the council is as follows:

Party	A	B	C
Number of council members	10	4	3

Using combinations to calculate the requested chance

Let us denote by S the number of corrupted members of political party B, i.e. members that accepted a bribe.
As there are only 4 members of party B in the town council, we can use

$$P(S \geq 2) = P(S = 2) + P(S = 3) + P(S = 4) \quad (1)$$

or

$$P(S \geq 2) = 1 - P(S = 0) - P(S = 1) \quad (2)$$

to calculate the requested chance.
If for example (2) is chosen, each of the chances $P(S = 0)$ and $P(S = 1)$ can be found based on the event of selecting 5 people out of 17. This can happen in C_{17}^5 ways (the number of combinations of 5 items selected from 17 different items). Among all those combinations there is a subset where $S = 0$. Then there are no corrupted members of party B, so they should all come from the other group, consisting of 13 members of party A or C.
So in $C_4^0 C_{13}^5$ of the cases we have that $S = 0$.
Thus

$$P(S = 0) = \frac{C_4^0 C_{13}^5}{C_{17}^5}.$$

Remember that $C_n^p = \frac{n!}{(n-p)!p!}$. $P(S=1)$ is found analogous to $P(S=0)$, where now one of the corrupted members will be chosen within the members of party B and the remaining 4 within the group of members of party A or C. After some arithmetic effort, we find that

$$P(S \geq 2) = 1 - P(S=0) - P(S=1) = 1 - \frac{C_4^0 C_{13}^5}{C_{17}^5} - \frac{C_4^1 C_{13}^4}{C_{17}^5} = 0.32984$$

We conclude that there is a chance of 33% that among the 5 corrupted council members there are at least 2 members of party B. This means that there is a chance of 2 out of 3 that at most 1 of the 4 members of party B in the council belongs to the corrupted ones. This news will damage the image of the party. This means that the party leader is in big trouble and should react.

Questions

1. Also the party leaders of party A and C are concerned about the honesty of their party members. Can you help them and calculate as well the chance that at least 2 of the corrupted members belong to their own party. For which party this chance will be the most? (A, B or C)
2. If there are only 4 corrupted members within the council, will the chance to have at least 2 members of party B among the corrupted members, be less or more than in the case where there are 5 corrupted members?
3. Which case will occur most frequently: 0 members of party B are corrupted, 1 member of party B is corrupted, 2 members of party B are corrupted, …, 4 members of party B are corrupted? (Assume that within the city council there are 5 corrupted members.)

Activity

Use 17 cards, 10 with the letter A, 4 with the letter B and 3 with the letter C. Choose 5 out of them without seeing the letter. Do this 30 times and count the number of times you have at least two cards with the letter B among the set of 5 cards. Let's denote this number by Y. Can you see that $\frac{Y}{30}100\%$ estimates the chance that at least 2 of the corrupted members belong to party B?