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How Far Can You See?

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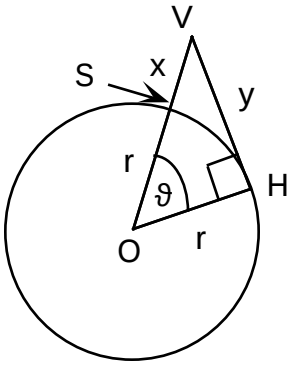
Problem: Fire towers are built so firefighters in forested regions can scan large areas for potential trouble. Upper-story and penthouse apartments rent for the highest prices because of the terrific views. Assuming the atmosphere is clear, and there are no hills or mountains obstructing the view, how far could you see to the horizon from the top of a 100 meter fire tower? From the top of a 55 story skyscraper, where each floor is 3 meters high? From the top of the tallest building in the world (the Petronis Tower in Kuala Lumpur, Malaysia, 1,467 ft. high)? From an airplane flying at 35,000 feet? From the Space Shuttle flying at 300 km?

Discussion: To do this problem, you need the radius of the Earth (r), which is about 6378 km (3963 miles). The viewer is at (V), the viewer's horizon is at (H), the point on the Earth's surface directly below the viewer is (S), and the center of the Earth is (O). (x) is the height of the viewer above the Earth, (y) is the distance along the line of sight, and OHV is a right triangle.



(not to scale)

$$\cos(\vartheta) = r / (r + x) \qquad (1)$$

$$\vartheta = \cos^{-1}(r / (r + x)) \qquad (2)$$

$$\underline{SH} = r\,\vartheta \qquad (3) \quad [\vartheta \text{ in radians}]$$

[Note: If (x) is very small compared to (r) -- which it usually is -- then $\cos(\vartheta) \sim (1 - \vartheta^2/2)$, and from (1) and (3), you can solve for $\underline{SH}$ using only algebra. $\underline{SH} \sim (2rx)^{1/2}$]

Answers: Tower -- 35.7 km, Airplane -- 369 km, Space Shuttle -- 1,919 km.

In the real atmosphere, other factors limit the visibility of distant objects. From a tall building on a clear day, you can see mountains as far away as about 100 miles.

Another suggestion: If your school is in a fairly flat area, you might try this: Do the calculation for the top of the school. You need to figure out the observer's height, either by sighting from the ground and using trigonometry, or by measuring the height of one story and multiplying. Then, have the class go up and determine the farthest objects they can actually see...