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Confidence in Medical Tests

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Medical tests are not perfectly accurate. Sometimes a test will indicate that the patient has a condition when he or she doesn't have it; this is called a "false positive". At other times the test will indicate that the patient doesn't have that condition when he or she really has it; this is called a "false negative". When a doctor and a patient receive the results of such a medical test, what is the probability that the result is correct?

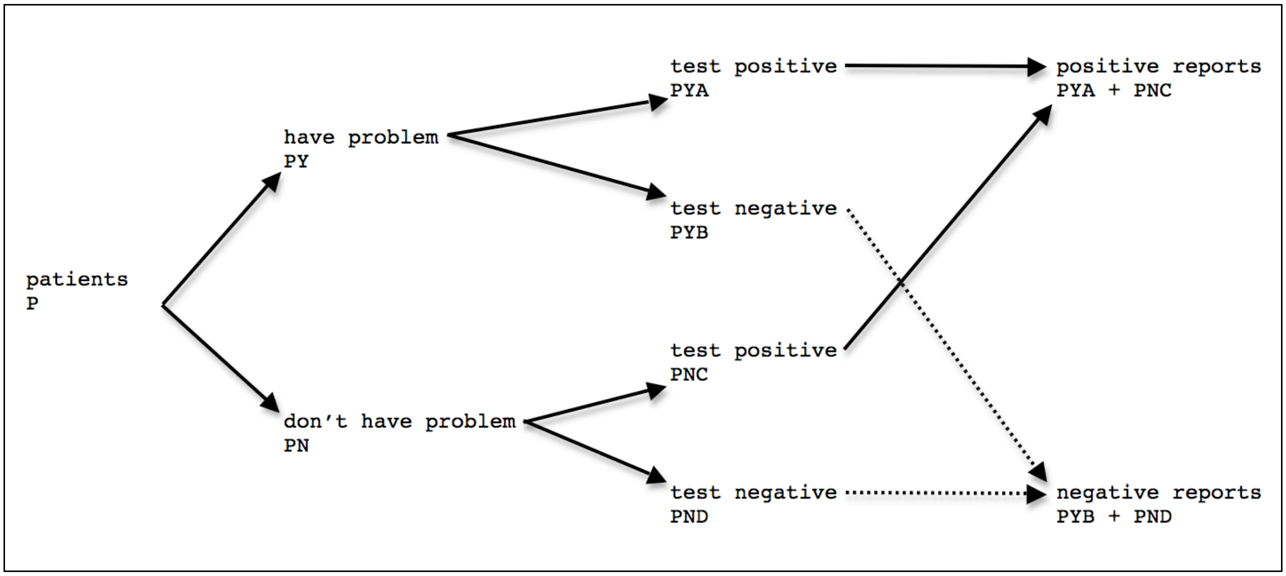
Analysis

The usual description of a medical test is similar to the following. Of people tested who have the problem, some percent will test positive (this is called "sensitivity"). Of people tested who don't have the problem, some other percent will test negative (this is called "specificity").

How can we adapt such a description in order to answer the question above?

Define variables as follows: Y fraction of people have the problem and N fraction of people don't. Of those who have the problem, A fraction will test positive (sensitivity) and B fraction will test negative (false negatives). Of those who don't have the problem, C fraction will test positive (false positives) and D fraction will test negative (specificity).

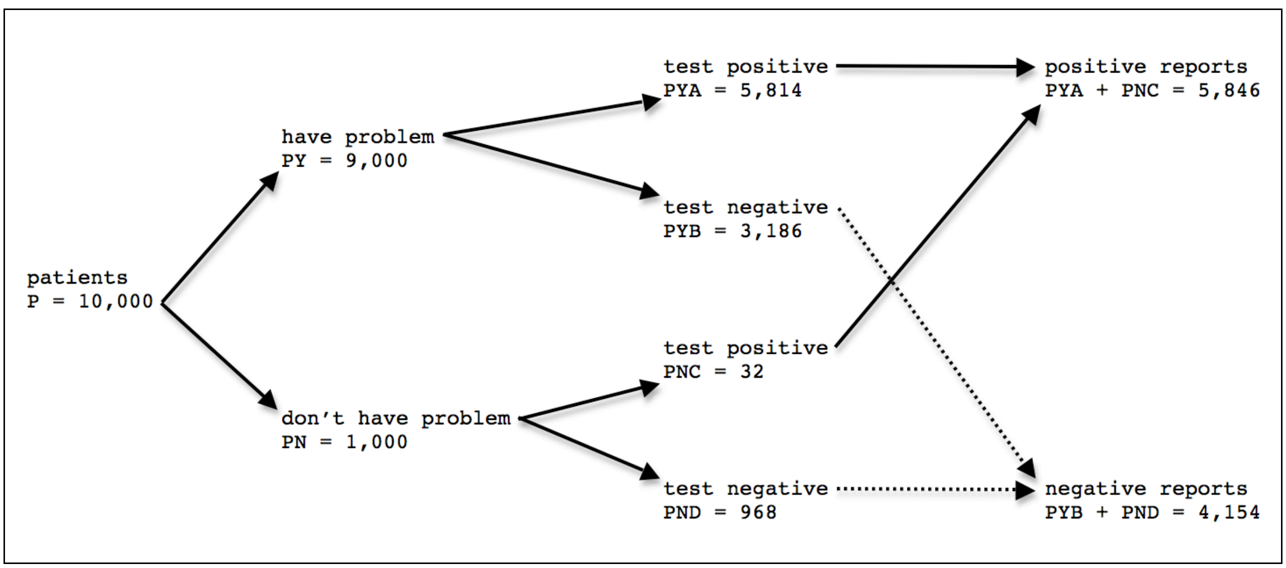
If we have P people in the population, then PY of them have the problem and PN don't. Of the PY patients with the problem, PYA of them will test positive and PYB will test negative. Of the PN patients who don't have the problem, PNC of them will test positive and PND will test negative. The number of positive tests and the number of negative tests are shown below.



So the probability that a person who tests positive really <u>is</u> positive is PYA / (PYA + PNC) and the probability that a person who tests negative really <u>is</u> negative is PND / (PYB + PND).

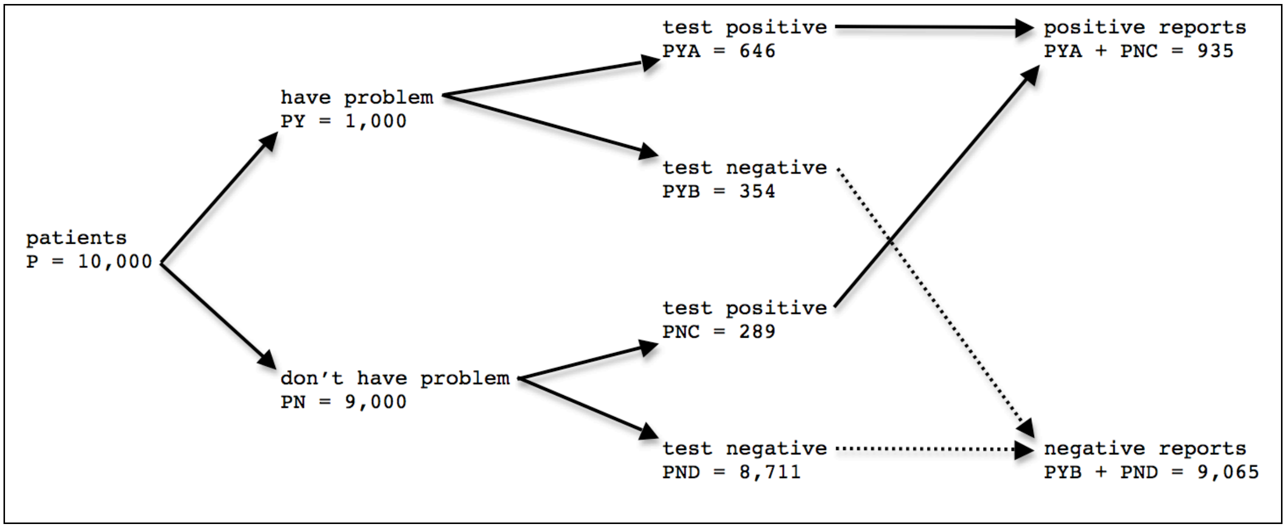
Numeric Examples

Example 1:
population P = 10,000   prevalence Y = 90%   sensitivity A = 0.6460   specificity D = 0.9679



The probability that a patient with a positive test really has the disease is 5814/5846 = 99% and the probability that a patient with a negative test really doesn't is 968/4154 = 23%.

Example 2: as above, but change prevalence Y to 10%.

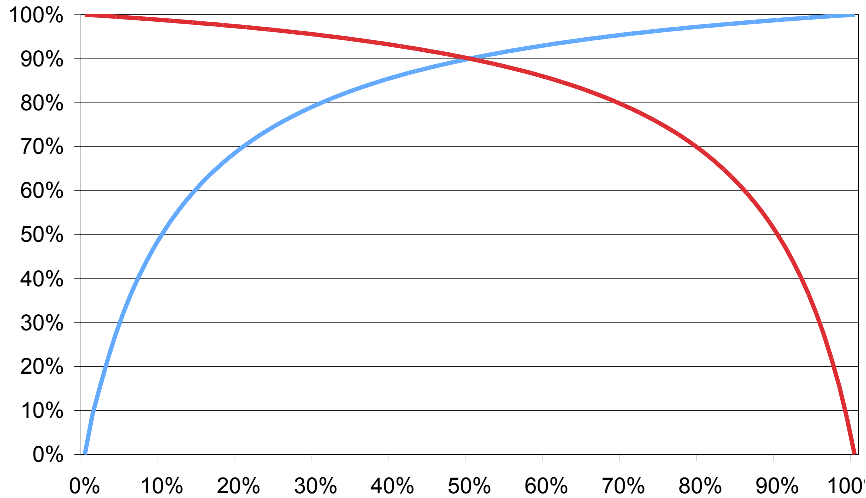


The probability that a patient with a positive test really has the disease is 646/935 = 69% and the probability that a patient with a negative test really doesn't have the disease is 8711/9065 = 96%.

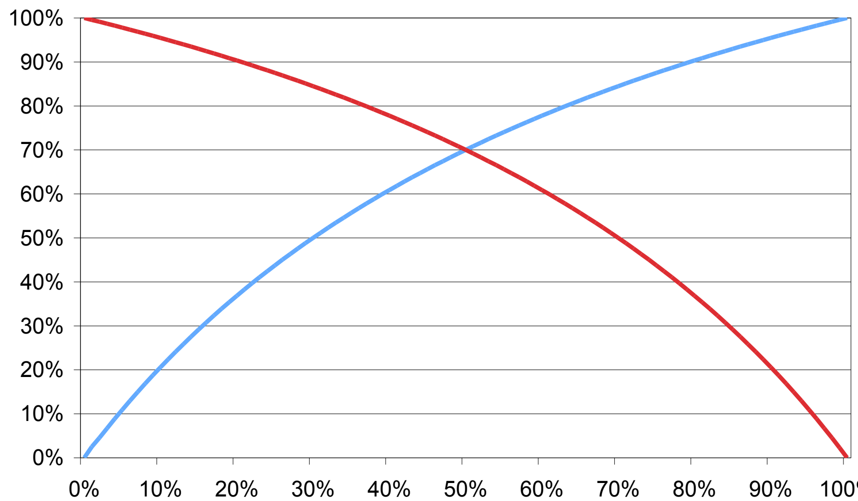
These examples show that confidence in test results depends on the prevalence of the problem, as well as the characteristics of the test itself.

Variables' Effects on Confidence

The next two graphs show how the prevalence of the condition in the population affects the probability that a positive report is correct and the probability that a negative report is correct.



X-axis: prevalence of the problem in the population
Y-axis (blue): probability that a positive report is correct
Y-axis (red): probability that a negative report is correct
When: probability of a false positive is 10% and probability of a false negative is 10%.

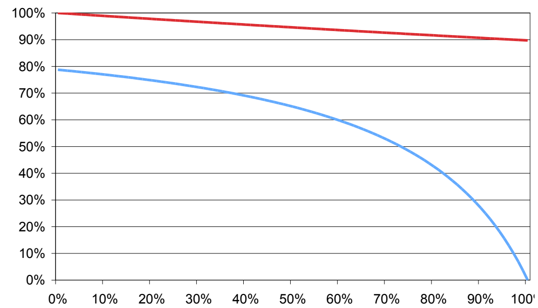


X-axis: prevalence of the problem in the population
Y-axis (blue): probability that a positive report is correct
Y-axis (red): probability that a negative report is correct
When: probability of a false positive is 30% and probability of a false negative is 30%.

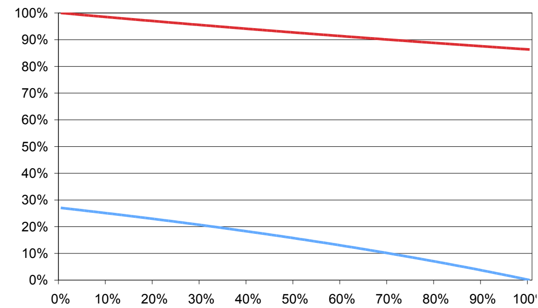
Those graphs illustrate the following relationships among variables:

1. Greater prevalence of the problem in the population increases confidence in positive reports and decreases confidence in negative reports.
2. Larger rates of false positives decrease confidence in positive reports and in negative reports.
3. Larger rates of false negatives decrease confidence in positive reports and in negative reports.

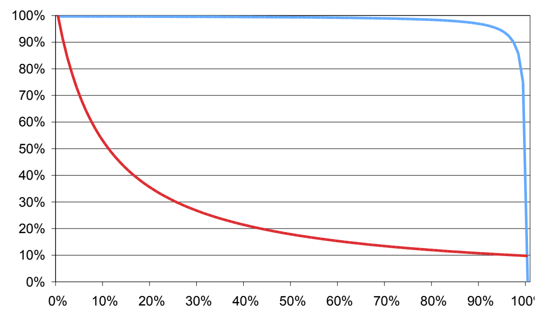
The next four graphs show how the rate of false negatives (shown on X-axes) affects the probability that a positive report is correct and the probability that a negative report is correct.



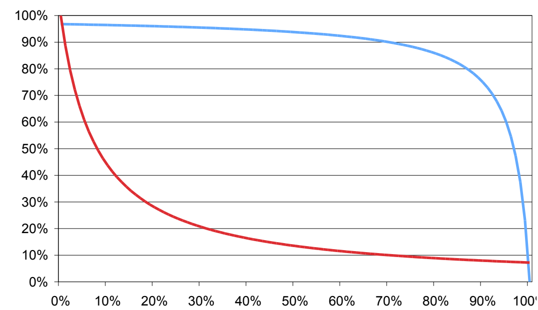
Prev = 10% False-pos = 3%



Prev = 10% False-pos = 30%



Prev = 90% False-pos = 3%



Prev = 90% False-pos = 30%

All four graphs:
X-axis: rate of false negatives
Y-axis (blue): probability that a positive report is correct
Y-axis (red): probability that a negative report is correct
When: prevalence of the problem in the population and
rate of false positives are as shown under each graph.

Those graphs illustrate the same relationships among variables as do the first two graphs.

Effects of Multiple Tests

If a physician needs more confidence about a patient's condition than is justified by the characteristics of a test and its reports, other tests might be ordered. If all tests report the same result, then confidence increases significantly. If the tests report mixed results, then additional tests can be ordered if they exist, or the physician can order treatment for the most serious condition that might exist.

Example 3: Suppose a doctor suspects the presence of a disease which can be fatal if not treated. Also suppose there are three different tests for the existence of that disease.

Test	Probability of false positive	Probability of false negative
A	0.20	0.27
B	0.45	0.18
C	0.38	0.30

If all three tests come back negative, what is the probability that the patient really has the disease? (That is, what is the probability that all three reports are wrong?) The probability of several independent conditions being true at the same time is the product of the probabilities of those conditions individually. So the probability that this patient has this illness is 0.27 x 0.18 x 0.30 = 0.01458 or about 1½ percent.

Do More Tests, or Not?

Those calculations show why physicians sometimes want more tests; they greatly increase confidence in the results. Extra tests cost extra money, so medical insurance companies want to avoid more tests. Sometimes insurance reimbursement decisions are based on financial ideas like these:

> Might the extra cost of an additional test lead to the choice of a less expensive treatment? Might the test save us more than it might cost us? Then we want to pay for that test; otherwise we don't.

Regardless of the possible costs involved, those financial ideas sometimes must yield to medical considerations like the following (although usually much less extreme in the consequences):

> A doctor believes a patient has either Disease-A or Disease-B, with no other alternatives, but is not sure which disease is present. The accepted treatment for Disease-A will kill the patient if he or she has Disease-B. The accepted treatment for Disease-B will kill the patient if he or she has Disease-A. Do as many tests as necessary to determine which disease exists.