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Square Roots Using a Carpenter's Square

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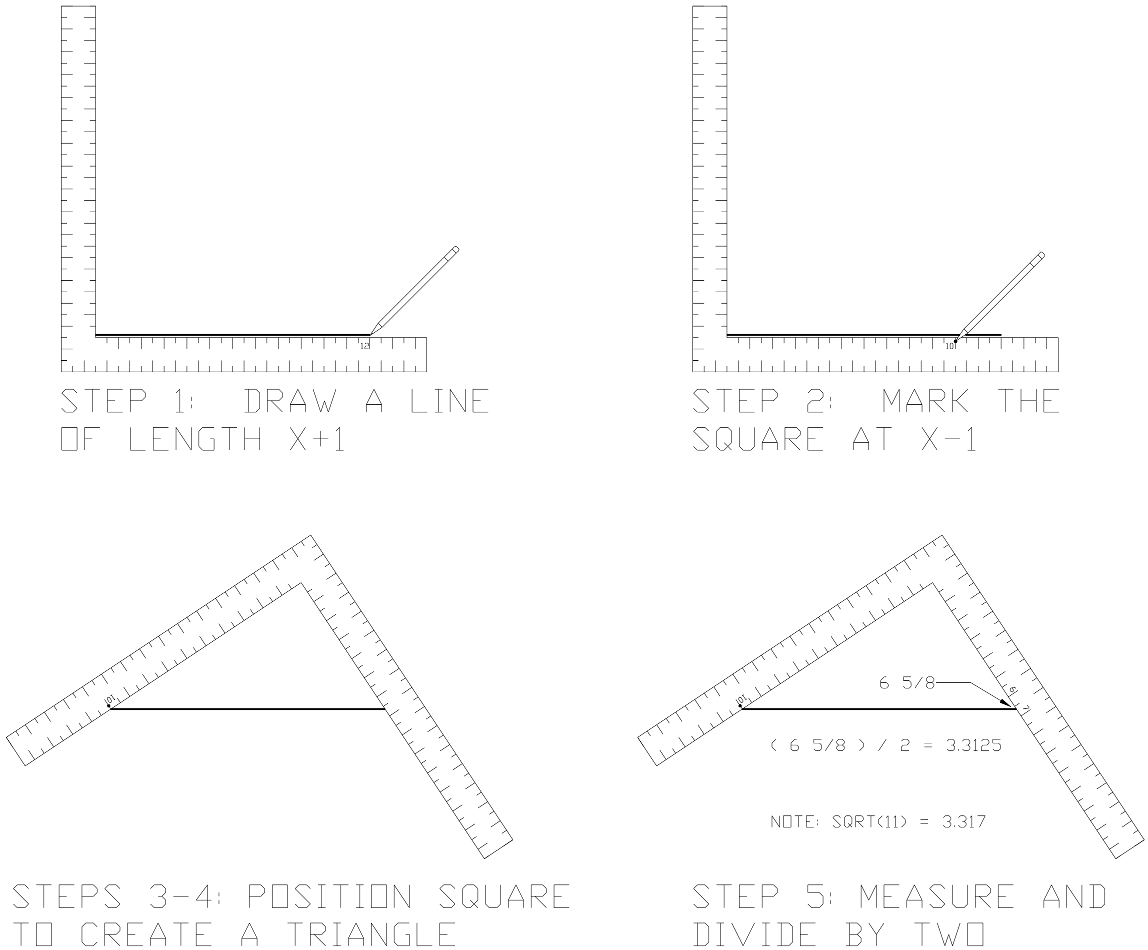
Background: As demonstrated in a PUMAS example from Lin H Chambers, "How Now, Pythagoras", master carpenters regularly make practical use of geometry and, at times, the Pythagorian theorem in their craft. I found this out some time ago when a good friend told me how an old-time carpenter he worked with calculated square roots using a carpenter's square. This carpenter performed his craft long before calculators arrived on the scene and, I'm sure, applied this method to numerous applications where he needed to ensure square corners or needed the length of a diagonal prior to cutting materials for construction. Being a mechanical engineering student at the time I learned this method, I was impressed by the elegant simplicity of it. I'm still impressed.

Method: To calculate the square root of a number, X, you will need a carpenter's square, a pencil, and a flat surface:

1) Draw a straight line of length X+1 inches on a flat surface using the carpenters' square. This will become the hypotenuse of the triangle you are constructing. If X is large, divide X by a square number such as 4, 9, 25, 100, etc. and use the result of this division as your X. As a final step you will multiply the result by the root of that square number to determine the square root of your original X. For example, if you divided by 64 you would multiply by 8 at the end.
2) Mark X-1 inches on one leg of the square (on the square itself). This will be one leg of the triangle you are constructing.
3) Place the square so the mark is on one end of the line drawn in step 1.
4) Rotate the square until the other leg of the square touches the other end of the line from step 1, keeping the mark from step 2 on the other end of the line.
5) Take down the measurement on that leg of the square and divide by two. If you divided the original number by a square number (64 for example), multiply the result by the root of that number (8 for example). The result is the square root of X.



DEMONSTRATION: SQUARE ROOT OF 11

Theory: The method relies entirely on the Pythagorean theorem:

$$
\begin{aligned}
(\text{leg two})^2 &= (\text{hypotenuse})^2 - (\text{leg one})^2 \\
(\text{leg two})^2 &= (X+1)^2 - (X-1)^2 \\
(\text{leg two})^2 &= (X^2 + 2X + 1) - (X^2 - 2X + 1) \\
(\text{leg two})^2 &= 4X \\
\text{leg two} &= \text{sqrt}(4X) \\
\text{leg two} &= 2\text{sqrt}(X) \\
&\text{or} \\
\text{sqrt}(X) &= (1/2)\ (\text{leg two})
\end{aligned}
$$

Example 1: You have been asked to construct a square table with a surface area of 15 square feet. What is the length of each side of this tabletop?

1) The area of a square tabletop is the square of the sides:

Area = Side * Side

or

Side = Sqrt(Area)

so

Side = Sqrt(15)

2) On a flat surface, draw a straight line of 16 inches. On one leg of the square, make a mark at 14 inches. Using the method outlined above, the resulting measurement on the second leg of the square will be about 7 3/4 inches.
3) Dividing by two gives us 3.875 as our square root, so each edge is 3.875 feet. (3.873 is the result using a calculator).

Example 2: A rocket lifts off from Earth, accelerating at a constant rate of 5 ft/sec^2 until it is 100 miles (528,000 ft) above Earth. How long did it take to reach this altitude assuming it traveled straight up?

1) For this example (with constant acceleration) we can use the following to solve for time:

   Distance = Acceleration * Time^2

   or

   Time = Sqrt(Distance/Acceleration)

   so

   Time = Sqrt(528,000 ft / 5 ft/sec^2)

   Time = Sqrt(105,620) seconds

2) 105,600 is a large number for using the carpenter's square so we will first divide by 10,000 (100^2) and calculate the square root of 10.560. We will also round to the nearest 1/16th of an inch, which is 10 9/16 inches (10.563 inches)
3) On a flat surface, draw a straight line of 11 9/16 inches. On one leg of the square, make a mark at 9 9/16 inches. Using the method outlined above, the resulting measurement on the second leg of the square will be about 6 8/16 inches (6.50 inches).
4) Dividing by two gives us 3.25 as our square root. Multiplying by 100 results in a time of 325 seconds (324.96 is the result using a calculator).

Final Notes: The accuracy of this method is a function of rounding errors, the accuracy and precision of the carpenter's square, and the skill in performing the steps. I have not analyzed the theoretical accuracy of this method (i.e., not counting human error or problems with the carpenter's square) and invite anyone to do so.