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Pilots, Airplanes, and the Tangent of Three (3) Degrees

By: Charles R. “Chuck” Moren, Associate Professor of Aeronautical Science,
Embry-Riddle Aeronautical University, Daytona Beach, Florida 32114.
e-mail: morenc@cts.db.erau.edu

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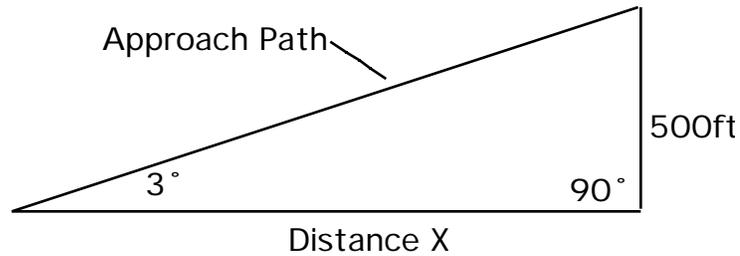
Objective: To show a practical application of Trigonometry in the aviation environment.

Equipment required: A scientific calculator with TAN function.

Background: When approaching an airport, pilots must learn to maneuver their aircraft visually, so that a stabilized approach to the runway can be flown at a constant approach angle. Precise approach planning insures a smooth transition to a landing within the Touchdown Zone (1) of the runway. Pilots must sometimes execute visual approaches that are varied in size, shape, and angle based upon a variety of factors such as: other aircraft, obstructions, noise abatement, or prevailing weather conditions. The standard approach angle however, is 3° . This 3° angle is the standard approach angle integrated into Instrument Landing Systems (ILS) (2) and Visual Approach Slope Indicators (VASI) (3) installed at many airports. When flying a normal traffic pattern, the aircraft is maneuvered so that the final approach is intercepted at an altitude of 500 feet above the elevation of the airport (AFE) (4).

Practical Application: When initially *teaching* pilots to execute standard visual approaches, instructors start in the *classroom* with a blackboard, chalk, and basic TRIGONOMETRY. Student pilots must visualize both their flight path over the ground (in reference to the runway) as well as their vertical approach path. In order for students to accurately visualize their ground path, they must first compute the distance they must be from the end of the runway when they turn onto their final approach segment (5).

Question: In order to fly the standard 3° approach, how long should the final approach segment be if the pilot plans to turn final at 500 feet AFE?



If we begin our final approach 500 feet AFE, we can compute the length of our final approach (Distance X) by dividing our altitude (500 feet AFE) by the tangent of 3° . So, $500 \text{ feet} / \tan 3^\circ = 9,541 \text{ feet}$.

$$\tan 3^\circ = \frac{500 \text{ feet}}{x}, \text{ thus}$$

$$x = \frac{500 \text{ feet}}{\tan 3^\circ}$$

To compute the proper position to begin our final approach in the classroom, we use our scientific calculator's TAN function.

(Before you begin, be sure that your calculator is in "degrees" mode.)

PRESS: 500 / 3 tan = (and your calculator should display) 9,540.5683 feet

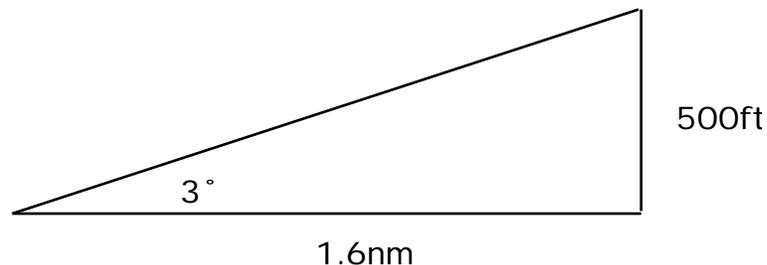
Answer: 9,540.6 feet.

If we turn onto the final approach at 500 feet AFE, 9540.5683 feet from our point of intended landing, and draw an imaginary line from the aircraft to the runway, we will have established our ideal (and imaginary) approach path.

A Conversion for Convention: Pilots are accustomed to judging distance in miles, more precisely Nautical Miles (nm).

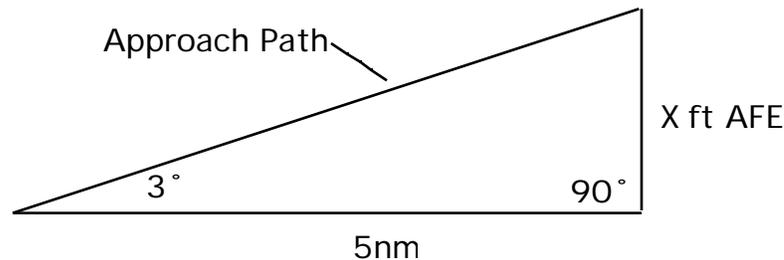
To convert 9,540.5683 feet to nautical miles, we use the factor 6076.1 feet per nm.

$9,540.5683 \text{ feet} / 6076.1 = 1.57 \text{ nm}$, or a final approach segment approximately 1.6nm in length.



Extension: Sometimes the length of the final approach is known and the pilot must determine the proper altitude to turn final.

Question: If the pilot planned the approach to begin 5nm (a typical distance for an ILS) from the end of the runway, at what altitude (AFE) should the aircraft be when intercepting the final approach?



$$\begin{aligned}1\text{nm} &= 6,076.1 \text{ feet} \\5\text{nm} &= 6076.1 \times 5 = 30,380.5 \text{ feet} \\30,380.5 \times 3 \tan &= 1,592 \text{ feet AFE}\end{aligned}$$

Answer: The pilot should maneuver the aircraft so as to turn final approach 5nm from the runway at an altitude of 1,592 feet AFE.

More Pilot Stuff (Pilot Math): Pilots use “rules-of-thumb” and printed tables (all based upon mathematical concepts) to estimate other important performance variables related to the operation of their aircraft.

For example...descent angle is a function of airspeed and rate of descent. Once the pilot has established the aircraft on a 3 degree approach “what rate of descent (in feet per minute) must be flown to maintain the 3 degree approach”? The variable is ground speed. Ground speed is the airspeed of the aircraft in Knots, plus or minus the headwind or tailwind component. A rule-of-thumb used by pilots is: “Airspeed multiplied by 5 equals the rate of descent (in feet per minute) required to maintain a 3 degree approach. In other words, an aircraft approaching at 100 knots, in a no-wind condition, must descend at 500 feet per minute to maintain a 3 degree approach path.

One more...pilots are required to make all turns in “instrument conditions” at 3 degrees per second. This is referred to in aviation as a “standard rate turn.” Rate of turn is a function of airspeed and bank angle. To determine the approximate bank angle required to establish a standard rate turn, the pilot uses the rule-of-thumb: airspeed divided by 100 multiplied by 1.5. A pilot flying at 120 knot would roll into at bank of 18 degrees in order to establish a standard rate turn.

Notes: Pilots do not fly aircraft with calculators in their laps. Classroom exercises are designed to ensure that the pilots understand the relationship of altitude and distance when flight planning in a 3 dimensional environment. In practical terms, a pilot who was flying an approach to a typical runway (that was 5,000 feet long) would estimate the length of the final approach segment by flying an approach that was approximately equal to two runway lengths.

1. **Touchdown Zone** - The first 3,000 feet of the runway.
2. **Instrument Landing System** - An electronic landing aid that transmits lateral and vertical (approach angle) guidance to the instruments in the aircraft.
3. **Visual Approach Slope Indicator** - A landing aid located at the end of the runway that provides approach angle guidance to the pilot via colored lights.
4. **Above Field Elevation** - The altitude of the aircraft above the elevation of the airport (field). Determined by the pilot by subtracting the airport elevation from the altitude displayed by the altimeter in the aircraft. The altimeter displays altitude in “mean sea level.”
5. **Final Approach Segment** - That part of the approach when the aircraft is aligned with the runway and is descending for landing.