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# Teaching Radicals in Less than Five Minutes

**By:** Juli Ratheal, West Texas A & M University, WTAMU Box 60787, Canyon, TX 79015
e-mail: jratheal@mail.wtamu.edu



Many students approach college level math courses with anxieties approaching phobic levels. Math instructors continue to search for innovative teaching methods which make math concepts easier to understand, and at the same time, lessen student fears.
Various techniques, such as mnemonics, acronyms and other minds/hands-on techniques, have been used in attempts to increase student understanding, encoding, retrieval and usage of information. This article presents a conceptual teaching tool which consists of attaching new ideas or techniques to a previously learned concept or behavior.

In psychology, one technique used to address fears is approximation, in which a person is confronted with a stimulus which invokes a small but similar anxiety to the anxiety being addressed. The level of dreaded stimuli is increased, until the anxiety-of- focus can be confronted by the person with a lowered or eliminated degree of fear. In an inverted fashion, the teaching technique presented here involves focusing on positive, previously learned concepts paired with concrete behaviors, and then transferring that knowledge to abstract themes, math concepts and problem solving.

### *The Problem*

Students often experience anxiety and fear when presented with mathematical symbols and variables, as well as the idea of understanding formulas and being confronted with a situation in which they must solve equations. One such situation involves the process of simplifying radicals, which is sometimes perceived by students to be a complicated and confusing problem. The technique presented in this article addresses these student concerns. The purposes of the following technique include (a) making connections between a difficult math concept and everyday game activities; (b) lessening student fears; and (c) enhancing student desires to learn mathematics. The technique has been field tested for over ten years by this instructor and has proven to be an effective way to get students' attention and of facilitating students' interest in learning to solve math problems once they grasp the overall concepts.

### *Technique for Simplifying Radicals*

The following technique for solving radicals is an effective tool for every mathematical root; for example, squares, cubes, and every variable above. The basic idea involves a process of sorting, a simple concept learned in kindergarten. The technique can be demonstrated by a number of well-known card games, including Old Maid, Go Fish, Yahtzee or Poker. In each case, the idea is to make a task perceived to be quite difficult, easy. Therefore, initial applications of the technique are limited to the use of natural numbers. Once the students have the process concretely visualized, the bar can be raised to include more advanced questioning prompts, such as, “Would including negative numbers require additional considerations?”

***Setting the Stage Activity***

In order to set the stage for learning complex math concepts, simple sorting procedures are demonstrated in eight steps, using ordinary playing cards. These are:

1. Distribute several decks of cards (with composite numbered cards removed), for every 3-4 students.
2. Instruct students to select a dealer and to deal everyone 10-15 cards each.
3. Students are to fan their cards in front of them face up.
4. Have the students make “books” of “two-of-a-kind” and instruct them to pull the cards out of their hand and place the “book” over to the left side of their hand.
5. Have several students describe how many “books” they made and pulled “out” of their hand and what cards are left in their hand and why.
6. Have students return all cards to the dealer and reshuffle the cards.
7. Students are dealt new cards but this time are instructed to make books of “three-of-a-kind.”
8. Repeat this activity with “four-of-a-kind,” “five-of-a-kind,” and so on.

***Transferring the Process to Mathematics***

The next step involves attaching the previously learned behavior of sorting, to the new technique of simplifying radicals. The following example demonstrates the process for facilitating such transfer:

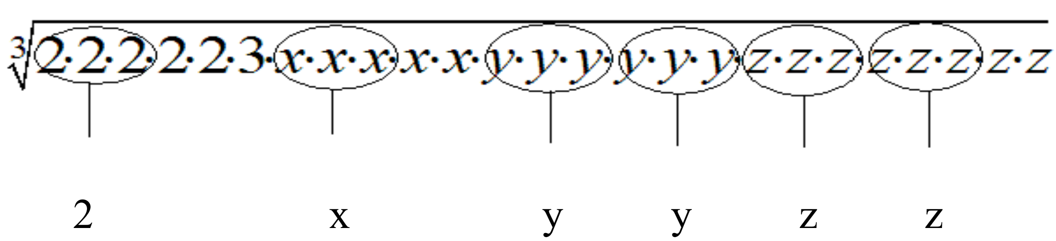
1. Write a radical on the board or overhead similar to the following:

$$\sqrt[3]{96x^5y^6z^8}$$

2. Have student expand the inside of the radical as follows similar to sorting cards in a hand:

$$\sqrt[3]{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 3\cdot x\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y\cdot z\cdot z\cdot z\cdot z\cdot z\cdot z\cdot z\cdot z}$$

3. Instruct students to circle all “three-of-a-kind” and emphasize that the “3” (root) simply indicates what size our “books” are to be.




4. Emphasize that these “books” are pulled out of the radical and placed over to the left, outside of the radical.

$$2xy^2z^2\sqrt[3]{2\cdot 2\cdot 3\cdot x\cdot x\cdot z\cdot z}$$

5. To finish, instruct students to simplify inside the radical.

$$2xy^2z^2\sqrt[3]{12x^2z^2}$$

5. Work several more examples utilizing other roots, emphasizing that radicals are no more complicated than playing cards such as Old Maid or Go Fish.

6. After several examples similar to the one above, write one on the board with exponents such as 1,024 which is sure to get their eyes opened wide.

7. After discussing that no one would want to write out that many variables, walk them through deriving their own rules as to what to do to solve this problem without having to resort to factoring the entire radical.

   Be sure and allow the students to "discover" that using division to determine how many books will be pulled out and how many remainders will be left inside the radical is an easy way to solve the problem.

***Conclusion***

Successfully learning to transfer the concept of simple sorting to the process of solving math problems may result in students feeling an excited sense of insight, understanding and accomplishment. In addition, this novel teaching technique provides a foundation on which practical applications and discussions involving higher order thinking skills can build once retention of the basic process of simplifying radicals is achieved. Within an academic discipline beset by fearful student expectations, one student's new experience of math as positively radical may also provide a rewarding experience for the instructor.

***Suggested Activity and Practical Applications***

In order to increase conceptual understanding and practice the process associated with simplifying radicals, the following activity is suggested. Divide the class into collaborative exploration groups. Provide each group with a graduated cylinder partially filled with water. Distribute non-floating, solid spheres of various sizes, such as glass marbles, to each group. Have students measure the volume of each sphere by the volume of water they displace, and document their findings. Then have the students calculate the diameter of each sphere by using the volume formula for a sphere ($V = \frac{4}{3}\pi r^3$). The students could then repeat the experiment, finding the volumes of irregular, non-floating objects, and the diameters of spheres having equivalent volume.

The following aircraft examples (Michael T. Mize, personal communication, March 1, 2007) are suggested in order to provide a framework from which other applications may be developed. Students are provided an opportunity to conduct hypothetical research related to specific aircraft and to design their own problems.

1. On landing rollout, the speeds (in ft/sec) at which a KingAir 200 Aircraft will hydroplane are greater or equal to the product of the square root of the airplane's tire pressure (in PSI = pounds per square inch) and 9. If the Ace of the Base, "Buck

Johnson," knows that his KingAir's tires are filled to 64 PSI, below what speed will Buck no longer have to worry about hydroplaning when he lands in rainy conditions later today?

Answer: $64 = 8^2$, so Min. Hydroplane speed $= 9 \bullet \sqrt{8^2} = 72$ ft/sec

Converting this to a possibly more familiar unit of miles/hour can be done with factoring as follows:

(72 ft/sec) • (3600 sec/hour) / (5280 ft/mile) =

$$\frac{(3^2 \bullet 2^3) \bullet (2^4 \bullet 3^2 \bullet 5^2)}{(2^5 \bullet 3 \bullet 5 \bullet 11)} = \frac{2^2 \bullet 3^3 \bullet 5}{11} = 49 \text{ miles / hour}$$

2. The prop-rotor disc is considered to be the disc or circle-shape formed by the path made by one complete rotation of a rotor blade's tip in its plane of rotation. The area of space that the prop-rotor disc of the V-22 Osprey requires to rotate is 324π sq. ft. What is the diameter of the prop-rotor disc?

$Area = \pi \bullet r^2 = 324 \bullet \pi$ So $r = \sqrt{324} = 18\,ft$., and diameter = 36 ft.

3. An auxiliary hydraulic system aboard a H-46 Sea-Knight helicopter can be filled by manually pumping a hand pump. The hand pump forces trapped *return* fluid (obtained from the 'up-stroke') from a cube-shaped container through a check-valve into the system reservoir (on the 'down stroke'). If Jake Reynolds pumps the hand pump 38 times resulting in a total of 1728 cubic inches of fluid being forced into the system reservoir, what is the side length of the cube-shaped container that initially holds the return fluid?

Answer: $\sqrt[3]{1728} = \sqrt[3]{2^6 \bullet 3^3} = 12$ in.