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Probability and Traffic Signals

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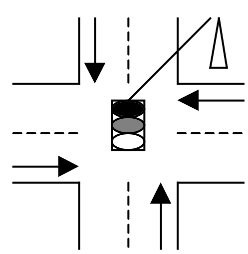
Introduction: It seems that traffic flow problems are becoming more and more common these days. In fact, despite the most noble efforts of city planners, even when there is very little traffic on the road, waiting seems to be a fact of life. So what is the probability that you will have to wait at a particular traffic signal, or perhaps more appropriately, what is the average amount of time you can plan on waiting? Let's look at a simple probability analysis related to traffic lights to find out.

A Simple Example: Suppose we are interested in predicting the average wait time W(avg) of a particular light for which the timing cycle is known. Let us assume for the purpose of simplicity that this traffic signal is at a 4-way intersection and that it is green for the same amount of time in each direction.

- The total time (measured in seconds) it takes to cycle through all four directions of the signal can be defined as c(t).
- The time the signal is green in any given direction (also measured in seconds) can be defined as d(t).

We could estimate roughly the probability of arriving at the light when it is already green for our direction as follows: P(no wait) = d(t)/c(t). However, because we are actually looking for the *average* amount of time we can expect to wait, we need to find a way to average all of the possible wait times. Let us attempt to define the intervals of waiting by using seconds as our interval:

- If the light is already green, the wait is 0 seconds
- If the light is red we can expect to wait anywhere from 1 second to c(t) - d(t) seconds
- Let us also associate each second a car could arrive at the intersection as an interval of 1/c(t) seconds or one fraction of the entire cycle



Because the number of seconds we could expect to spend waiting at the light is associated with a discrete integer value, and is incrementing by units of one second at a time, the average number of seconds spent waiting can be calculated using the sum of consecutive integers formula divided by c(t), the total number of one-second intervals in the cycle. In essence, adding up all the possible second values you may have to wait and then dividing by the total. Consider the following derivation:

{1 + 2 +… + [c(t)-d(t)-k] +… + [c(t)-d(t)-2] + [c(t)-d(t)-1] + [c(t)-d(t)]**}** all divided by c(t)

Substituting into the formula Σ i = n(n+1)/2 and dividing by c(t)we get the following: W(avg) = **{**[c(t)-d(t)]*[c(t)-d(t)+1]**}**/2c(t)

Let us now test our average wait time model for a known cycle. Suppose the complete cycle of our signal is 60 seconds, and the light is green for 15 seconds in each direction.

For this example we set c(t) = 60 and d(t) = 15. Substituting into our model gives us the following:

- W(avg) = **{**[c(t)-d(t)]*[c(t)-d(t)+1]**}**/2c(t)
- W(avg) = **{**[60-15]*[60-15+1]**}**/2(60)
- W(avg) = (45)(46)/120
- W(avg) = 17.25 seconds

From this calculation, we can conclude that the average time we will have to wait at the intersection is 17.25 seconds. Of course many other variables would come into play in a more realistic setting, from the number of cars in front of us to the speed we drive between the lights. However, the simple example does give us a sense of how to build a mathematical model using a very commonly taught formula (sum of first n integers) to solve a rather practical problem.