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Motivating Honest Budgets and Hard Work Simultaneously

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Some problems in the real world can be solved by the use of mathematical ideas and reasoning without solving any formulas or doing any calculations. Here is an example.

(The problem, its solution, and a numeric example are shown immediately below. Some suggestions for use as a thought-provoking classroom exercise then follow. Teachers are invited to describe other successful implementations to 4thecho@gmail.com.)

Motivating managers to send accurate budget/forecast estimates to their bosses is not always easy. Some managers will submit overly optimistic budgets/forecasts, hoping to look good to their bosses. Other managers will turn in pessimistic budgets/forecasts, confident that they will be able to “beat the budget” and earn bonuses accordingly.

Motivating hard work is not always easy either. Most people believe that higher pay stimulates harder work. But, once a fixed pay rate is agreed upon, that pay rate by itself does not motivate harder work. A pay scheme motivates harder work if and only if better results lead to more pay.

However, we can motivate the subordinate manager to submit an honest budget, and then work hard after the budget is submitted, by using the contractual pay plan described below. Such bonus contracts are common in businesses, especially in organizations that are so large that the top executives believe they cannot supervise all their subordinate managers in sufficient detail or sufficiently often to guarantee total compliance with all desired management behaviors. Therefore these bonus contracts are used as partial substitutes for such supervision. Since these payments are contractual, they are an obligation the business must pay to involved individuals regardless of the size or existence of company profits.

1. Make the manager's bonus \mathbf{B} some percentage \mathbf{b} of budgeted profit \mathbf{P}_{BUD} : $\mathbf{B} = \mathbf{bP}_{\text{BUD}}$. This motivates the manager to make the budgeted profit large.

2. If actual profit \mathbf{P}_{ACT} is less than budgeted profit, subtract from the bonus some percentage \mathbf{c} of the profit shortfall: $\mathbf{B} = \mathbf{bP}_{\text{BUD}} - \mathbf{c(P}_{\text{BUD}} - \mathbf{P}_{\text{ACT}})$ if $\mathbf{P}_{\text{ACT}} < \mathbf{P}_{\text{BUD}}$. This provides a disincentive for the manager to make the budgeted profit too large. It also motivates hard work after the budget is set.

3. If actual profit is more than budgeted profit, add to the bonus some percentage \mathbf{a} of the profit surplus: $\mathbf{B} = \mathbf{bP}_{\text{BUD}} + \mathbf{a(P}_{\text{ACT}} - \mathbf{P}_{\text{BUD}})$ if $\mathbf{P}_{\text{ACT}} > \mathbf{P}_{\text{BUD}}$. This provides an incentive for the manager to work hard after the budget is set.

4. Choose \mathbf{a} , \mathbf{b} , and \mathbf{c} such that $\mathbf{0} < \mathbf{a} < \mathbf{b} < \mathbf{c} < \mathbf{1}$ and such that the differences between \mathbf{c} and \mathbf{b} , between \mathbf{b} and \mathbf{a} , and between \mathbf{a} and $\mathbf{0}$ all are large enough to make a significant impact on the manager's perceptions and behavior.

(If \mathbf{a} is less than $\mathbf{0}$ the manager is being penalized for creating profits - definitely not motivational. If \mathbf{a} equals $\mathbf{0}$, the manager gets no bonus and the plan doesn't motivate the desired results. If \mathbf{c} equals $\mathbf{1}$ the manager gets all the profits and the company gets none, violating the reason for its existence. If \mathbf{c} is greater than $\mathbf{1}$ the company is giving the manager more value than (s)he created for the company; the company is helping the employee raid the treasury.)

(Laboratory experiments and experience in real businesses suggest that differences of about thirty percent are adequate for these purposes. As long as each threshold is met or exceeded, setting the actual numeric values of these parameters is a matter of executive judgment, policy, and preference.)

Then the complete bonus plan is:

$$\begin{aligned} \mathbf{P}_{\text{ACT}} > \mathbf{P}_{\text{BUD}} &\Rightarrow \mathbf{B} = \mathbf{b P}_{\text{BUD}} + \mathbf{a (P}_{\text{ACT}} - \mathbf{P}_{\text{BUD}})} \\ \mathbf{P}_{\text{ACT}} = \mathbf{P}_{\text{BUD}} &\Rightarrow \mathbf{B} = \mathbf{b P}_{\text{BUD}} \\ \mathbf{P}_{\text{ACT}} < \mathbf{P}_{\text{BUD}} &\Rightarrow \mathbf{B} = \mathbf{b P}_{\text{BUD}} - \mathbf{c (P}_{\text{BUD}} - \mathbf{P}_{\text{ACT}})} \end{aligned}$$

The manager cannot benefit from lowballing the budgeted profit to earn a bonus later, because the reward \mathbf{a} from beating the budget is significantly smaller than the reward \mathbf{b} from increasing the original budgeted profit. The manager cannot benefit from inflating the budgeted profit to look good to a higher-level boss, because the penalty \mathbf{c} from failing to meet the budget is significantly larger than the reward \mathbf{b} from increasing the original budgeted profit. So the manager is motivated to create an honest budget. The manager benefits from working hard

because **a** is greater than zero. Therefore, this pay plan motivates honest budgets and hard work simultaneously.

Here is a numeric example. Suppose a subordinate manager submits a budget that predicts a profit of P_{BUD} , and suppose P_{BUD} is so large that one percent of it is a sufficient motivator for that manager.

Then we could choose:

$$\begin{aligned} \mathbf{a} &= 0.01 P_{BUD} \\ \mathbf{b} &= 1.3 \times 0.01 P_{BUD} = 0.013 P_{BUD} \\ \mathbf{c} &= 1.3 \times 0.013 P_{BUD} = 0.0169 P_{BUD} \end{aligned}$$

(If other, larger, multipliers are chosen they should provide stronger incentives for the manager to act as desired. If any multiplier is chosen to be smaller than indicated here, the motivational scheme could be inadequate.) These choices of **a**, **b**, and **c** create the following bonus plan:

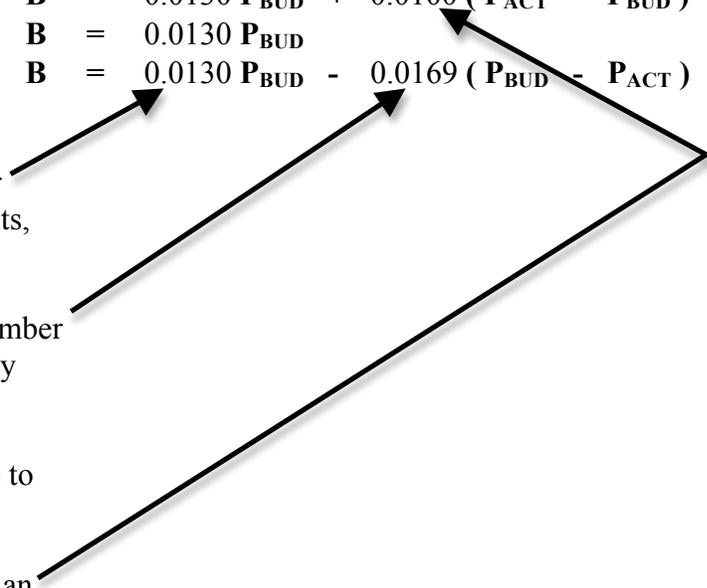
$$\begin{aligned} P_{ACT} > P_{BUD} &\Rightarrow B = 0.0130 P_{BUD} + 0.0100 (P_{ACT} - P_{BUD}) \\ P_{ACT} = P_{BUD} &\Rightarrow B = 0.0130 P_{BUD} \\ P_{ACT} < P_{BUD} &\Rightarrow B = 0.0130 P_{BUD} - 0.0169 (P_{BUD} - P_{ACT}) \end{aligned}$$

This number gives the manager an incentive to budget big profits, to get a significant reward

but not too big, because this number would then create a significantly larger penalty for failure.

So the result is strong incentive to submit an accurate budget.

And then this number provides an incentive for the manager to work hard, to get a significant reward for exceeding expectations.



SUGGESTIONS FOR THE CLASSROOM:

The following steps will lead students through the analytical processes that mathematicians use to solve problems. The more you can guide them to discover on their own, the better. The less specific help they need at each step, the better. Even if a class needs a lot of help, your explanations will lead them step-by-step through the thought processes involved and will help them to think about how they might analyze other problems on their own.

1. Describe the problem without giving any hints about the nature of the solution. Have the class brainstorm about what motivates business managers personally. (Theoretical economists usually assume people are highly motivated by money, never have enough, and don't care about anything else. Behavioral economists also consider that most real human beings value leisure, try to avoid risk, and have other motivations for their behaviors.)

2. Tell students that today's exercise focuses on money as the only motivator. (Adding behavioral economics considerations would be far beyond the purpose of today's class.) Ask the class how top executives might use money to encourage the behaviors they want their subordinate managers to demonstrate every day. For each idea suggested ask the class what happens to the manager at the end of the budget period if (s)he budgeted profits to be much more or much less than reality turned out to be. Then ask what happens at the end of that time if achieved profits are much more or much less than the budget that was established. We want the discussion to converge on the three components of the plan described above:

- A. The manager gets a share of the profits.
- B. The manager gets a larger share of profits if they exceed the budget
- C. The manager gets a smaller share of profits if they are smaller than the budget.

3. If the discussion doesn't converge as desired within a reasonable time, tell the students that the solution has those three characteristics. Show them the formulas that define the plan:

$$P_{ACT} > P_{BUD} \Rightarrow B = b P_{BUD} + a (P_{ACT} - P_{BUD})$$

$$P_{ACT} = P_{BUD} \Rightarrow B = b P_{BUD}$$

$$P_{ACT} < P_{BUD} \Rightarrow B = b P_{BUD} - c (P_{BUD} - P_{ACT})$$

4. Ask the class what relationships among **a**, **b**, and **c** would motivate the desired behaviors:

- A. Don't set the budget too high
- B. Don't set the budget too low
- C. Work hard so as to beat the budget that has been set

If the students have a hard time thinking about abstract relationships among those three variables, ask for numeric values that might be put into the equations. Calculating the results of a set of suggestions often leads people to adjust the values so that they have the proper relationships.

5. If the discussion does not converge to the correct relationships $\mathbf{a} < \mathbf{b} < \mathbf{c}$ then suggest either $\mathbf{a} < \mathbf{b}$ or $\mathbf{b} < \mathbf{c}$ or $\mathbf{a} < \mathbf{c}$ and see if the rest of the relationships become obvious to the group. Give the class all of the solution only as a last resort when time is up.