

Document ID: 09_22_16_2

Date Received: 2016-09-22 **Date Revised:** 2017-03-24 **Date Accepted:** 2017-03-26

Curriculum Topic Benchmarks: M1.4.2, M3.4.1, M7.4.10, M7.4.13

Grade Level: High School (9-12)

Subject Keywords conditional probability, Probability, expected value, value of information, risk, attitudes toward risk

An Example of the Value of Information

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In many economic situations, better information increases the probability of getting better results. That allows us to compute a monetary value of the improved information. If we can obtain that added information for a cost that is less than its value, we should do so. If the cost of the information equals its value, we gain nothing by acquiring the information. If the cost is greater than the value, we should not waste our money to get the information; we are better off economically to “go without”.

Expected Value

Understanding the value of information requires using the concept of expected value. Expected value is computed as follows:

1. List all possible outcomes of the situation.
2. For each outcome, multiply its financial result by the probability that the outcome will occur.
3. Total those calculated values.

For a real example, suppose you make a \$1.00 bet on a fair roulette wheel. A roulette wheel has numbers 1 through 36 plus 0 and 00. A winning number pays 36-to-1.

Your expected value of betting on any specific number is:

1. The \$1.00 you bet up front to buy into the experience is a guaranteed event. (It already happened. It's probability is 1.) So it's expected value is $-\$1.00 \times 1 = -\1.00 .
2. The ball lands on your number or it doesn't.
3. Your expected value of the winning number is $\$36.00 \times 1/38 = \0.95 .
4. Your expected value of not winning is $\$0.00 \times 37/38 = \0 .
5. Your total expected value of the bet is $-\$1.00 + \$0.95 + \$0.00 = -\0.05 .

The person who owns the wheel (known as “the house”) has this expected value:

1. With probability 1, the house pays \$0.00 up front to buy into the experience. So the expected value of that isolated event is $\$0.00 \times 1 = \0.00 .
2. The ball lands on your number or it doesn't.
3. The house's expected value of your number winning is $-\$36.00 \times 1/38 = -\0.95 .
4. The house's expected value of your number not winning is $\$1.00 \times 37/38 = \0.97 .
5. The house's total expected value of the bet is $\$0.00 - \$0.95 + \$0.97 = \0.02 .

So your total expected value of the complete experience is negative, but the house's is positive. That explains why roulette players usually go broke but casinos make profits.

The Oil Project

Now let's look at a fictitious, simplistic example that shows how information affects the expected value of an oil exploration project. Imagine that a large agricultural area, previously unexplored, recently has been the site of several oil discoveries. Suppose a family living on an average size farm in that area wants to get rich quick. Should they drill for oil, or not?

Let's imagine the following facts about the area:

1. 80% of exploratory wells have been dusters, finding no oil at all.
2. 15% of exploratory wells have brought in small strikes, producing \$2,000,000 worth of oil.
3. 5% of exploratory wells have brought in gushers, producing \$12,000,000 worth of oil.
4. Drilling an exploratory well costs \$1,000,000.

Then the expected value of drilling an exploratory well is

$$\$0 \times 0.80 + \$2,000,000 \times 0.15 + \$12,000,000 \times 0.05 - \$1,000,000 = -\$100,000.$$

That negative value tells us that merely "playing the odds" without investigating the farm's underlying geological structures is expected to be a losing proposition. The family should not drill.

Now suppose that the family learns of an exploration firm that does not belong to an existing oil company, but instead hires out their services to anybody who can pay their fees (in advance, of course; these people don't take risks as their customers do). They use sound-generating equipment to do a seismic survey of the rock structures under the land. They can determine for certain whether the subsurface rock structures include oil-bearing strata or not, but they cannot estimate how much oil might be present. This service costs \$200,000. Should the family hire those people or not?

The expected value of the entire project with use of the exploration firm:

Pay \$200,000 for the service. Thereafter:

1. There is an 80% chance that no oil-bearing strata were found. Stop the project.
2. There is a 15% chance of a small strike worth \$2,000,000 which cost \$1,000,000 to bring in.
3. There is a 5% chance of a gusher worth \$12,000,000 which cost \$1,000,000 to bring in.

So the expected value is:

80% chance of	-\$200,000
+ 15% chance of	-\$200,000 - \$1,000,000 + \$2,000,000
+ 5% chance of	-\$200,000 - \$1,000,000 + \$12,000,000
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	\$500,000

Therefore, the family should hire the independent exploration firm and then drill - or not drill - in accord with their findings.

If the firm charged \$700,000 for their services the family would gain nothing by hiring them. If they charged more than that, they definitely would not be hired. If they charge anything less than that, hiring them would give the family an economic advantage.

(Note that, to keep this example simple, additional factors that might influence such decisions have been excluded. Such factors include crop losses presently and in the future, environmental or aesthetic impacts of a drill rig and a permanent pump and storage tanks, and breaking with family tradition, among others. Some of those factors are rationally quantifiable but others are not.)

Real People and Risk

The recommendations above come from calculations which assume people are perfectly rational and care about only money. Real people aren't like that, because they demonstrate the following characteristics:

1. Most people are risk-averse, meaning that they would rather avoid risk whenever possible. This makes people less likely to accept some economically rational risks such as those described above.
2. Most people's perceptions and judgments about large wagers differ from those about small wagers. Many people will play double-or-nothing for a dollar based on a coin toss, but almost nobody will do that for their entire year's pay.
3. Most people attach more importance to a loss of a certain size than to a gain of the same size. A pay raise of 5% will cause some pleasure, but a pay cut of 5% will cause much more stress.
4. Most people are willing to risk only amounts they would not miss if they disappeared.
5. Most people react to a one-time risk differently than they react to many identical risks. That is because a large number of experiences tend toward the mathematically expected results as above, but a single experience will produce the dramatic extreme-case results.

All of these human characteristics affect economic decisions in the real world. Behavioral and experimental economists study how those traits change people's uses of rational assessments of expected value, thus altering economic reality. Many of those behaviors can be modeled in theory as simultaneous variables, quantified, predicted, and measured in society at large or in controlled experiments.

To gain a quick appreciation of those factors, imagine that you are the sole decision maker for your family. In your real life, would you spend \$200,000 up front for a project that had an 80% chance of stopping, so that you get nothing for that money?