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Proving the Pythagorean Theorem

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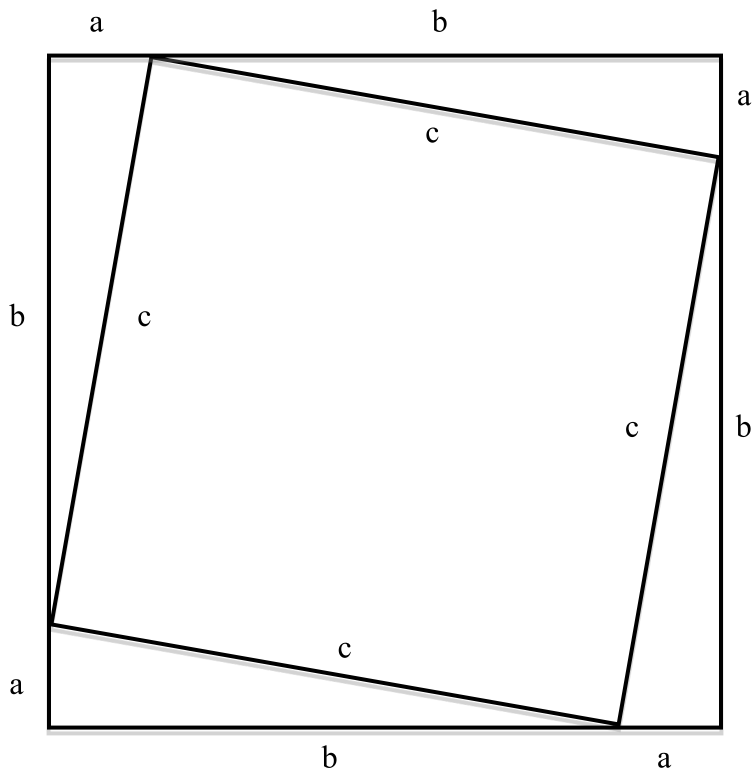
Part of math's value lies in its collection of algorithms that can be used to solve problems. Another component of math's value comes from a way of thinking that enables people to investigate new situations and problems, recognize patterns and relationships inherent in them, gain new insights, and perhaps develop appropriate new algorithms. Math students spend most of their time learning to solve problems by imitating the use of algorithms that have been described to and demonstrated for them. Much less time is devoted to development of creative thinking processes that can be applied to new situations and problems.

This paper describes a classroom exercise that contributes to the long-term development of creative thinking and analysis. It does so by directing students' attention to a subject without revealing the purposes of the tasks that will be suggested. Step by step, students are led through a reasoning process that leads to discovery of something they did not anticipate. This process gives students experience in analyzing a situation, and shows that an inquiry need not be purposeful initially in order to be productive eventually; sometimes just indulging one's curiosity can lead to beneficial discoveries.

The First Task

Assign each student to write a formula for the area of each region in this figure:



Each student should have created this list:

area of each triangle = $ab/2$

area of small square = c^2

area of large square = $(a + b)^2$

The Second Task

Tell students to write a formula that relates the area of the big square to the areas of all the regions it contains. Each student should see that

(area of big square) = (area of little square) + 4 x (area of a triangle)

so

$$(a + b)^2 = c^2 + 4(ab/2)$$

The Third Task

Have each student expand the parenthetic phrase and simplify the resulting formula. Their work should be similar to the following.

$$(a + b)^2 = c^2 + 4(ab/2)$$

$$a^2 + 2ab + b^2 = c^2 + 2ab$$

$$a^2 + b^2 = c^2$$

So?

When we look at that result, what do we see? The Pythagorean Theorem!

This experience is typical of mathematical ways of thinking and lines of inquiry:
1. Attention is drawn to a subject, whether by one's own curiosity or by another's direction.
2. Note what we know about the subject.
3. Look for relationships among ideas we have.
4. Deduce implications of that knowledge and those relationships.
5. Interpret those implications and report those that are informative and useful.

Using this process to apply existing knowledge to new subjects increases our understanding of mathematics and its applications to various fields of human inquiry. Greater knowledge of applied math leads to progress in the physical, biological, and social sciences as well as technology, engineering, manufacturing, transportation, economics, finance, information processing, and making decisions.